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NUMERICAL APPROXIMATIONS OF SELECTED
METEOROLOGICAL PARAMETERS RELATED
TO CLOUD PHYSICS

Walter S. Nordquist, Jr.

Army Electronics Command
White Sands Missile Range, New Mexico

March 1973

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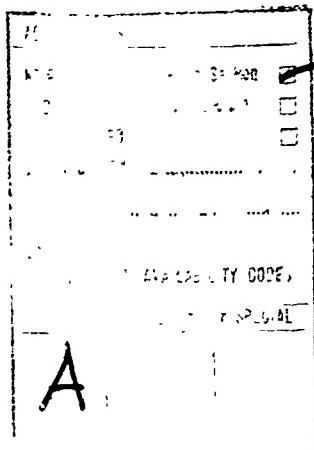
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NUMERICAL APPROXIMATIONS OF SELECTED
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ABSTRACT

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LIST OF SYMBOLS

<u>Symbol</u>	<u>Subscript</u>	<u>Item</u>	<u>Units Used</u>
c	r w	Specific Heat air at constant pressure liquid water	ITcal g ⁻¹ K ⁻¹
e		Water Drop Diameter	m
3	a s s _i s _w	Partial Pressure of Water Vapor actual saturation saturation over solid water saturation over liquid water saturation at the wet bulb temperature	mb
log		Common Logarithm (base 10)	
L	f s v	Latent Heat of water fusion sublimation vaporization	ITcal g ⁻¹
M		Mass of Water Drop	g
P	a t	Pressure atmosphere (with no water) atmosphere total	mb
R		Dry Air Gas Constant	ITcal g ⁻¹ K ⁻¹
rh		Relative Humidity	Percent
t		Time	Sec
T	a d LCL m s w x	Temperature dry bulb dew point lifting condensation level pseudo-adiabatic saturation-adiabatic wet bulb approximation of saturation- adiabatic	K
X	s w z	Mixing Ratio water vapor at saturation liquid water initial value of X _s	g g ⁻¹
ϵ		Ratio of molecular weights of water vapor and dry air	--

INTRODUCTION

The application of techniques for numerical simulation to various meteorological processes requires that many meteorological parameter descriptions be subject to individual approximation. Thus, the accuracy and adequacy of the descriptions are dependent on the requirements of the individual "simulator." In many cases the importance of the meteorological parameter description is subordinate to some broader scale problem; as such, if the description is deemed adequate, it is normally used without explanation or comment. As a consequence there are many methods available for the description of meteorological parameters, but there is a paucity of information on how well they describe the parameters. If the accuracy must be known, then the investigator must either construct his own approximation or ascertain the accuracy of some available approximation, either of which can detract from the solution of a larger problem.

This report contains descriptions of numerical approximating techniques for meteorological parameters required by the author and his associates in conjunction with numerical modeling efforts concerning clouds and fogs [1], [2], [3], [4]. This presentation is made so that there might be a ready reference to a set of approximating methods and the accuracies associated with the calculated parameter values.

The numerical approximations are divided into two groups: (1) descriptions of meteorological parameters, per se; and (2) descriptions of meteorological parameters associated with a meteorological process. The first group covers the latent heats of water, the saturation vapor pressures of water, the dew point and wet bulb temperatures, and the specific heat of liquid water. The second group covers the temperature of the lifting condensation level, the change of mass of freely falling water drops, and the temperatures resulting from saturated adiabatic processes. The method of approximation and the source of comparative values, as well as the error as determined relative to the comparative values, is provided for each parameter. The units for the comparisons are those of the comparative base, where applicable. In those cases where comparative values are not available, the basis of the accuracy given is discussed. When percentage errors are given the value presented is calculated from

$$\text{Percentage Error} = 100 \frac{\text{Calculated value}-\text{Reference value}}{\text{Reference value}} \quad (1)$$

The FORTRAN programming language listing of the approximations is provided in the Appendix in either FUNCTION STATEMENT or SUBROUTINE form as appropriate.

METEOROLOGICAL PARAMETERS

The meteorological parameters described in this section are associated with the state of the atmosphere and are, therefore, essentially definitions of the value of the parameter for the specified state. The parameters covered are those associated with the following: the heat requirements for the change of phase of water; the partial pressure of water vapor over liquid and solid plane water surfaces; the temperature at saturation of air cooled at constant pressure with no change in water vapor content; the temperature of air cooled to saturation at constant pressure; and the specific heat of liquid water.

Latent Heats

The latent heats are associated with the vapor-liquid, vapor-solid, and liquid-solid phase changes of water. The latent heats can be expressed as functions of temperature, T, with the techniques given here using degrees Kelvin (conversion to other temperature units is provided in Table I) and the latent heat expressed in ITcal g^{-1} (conversion to other units is provided in Table II).

The latent heat associated with the liquid-vapor/vapor-liquid phase changes is the latent heat of vaporization, L_v . It can be approximated from

$$L_v = 754.817 - 0.575 T \quad (2)$$

The latent heat associated with the solid-vapor/vapor-solid phase changes is the latent heat of sublimation, L_s . It can be approximated from

TABLE I¹
TEMPERATURE UNIT CONVERSIONS

Unit	Symbol	Unit	Symbol	Conversion
Kelvin	K	Centigrade	C	$K - 273.16$
		Fahrenheit	F	$\frac{1}{2}(K - 273.16) + 32$
		Rankine		$\frac{9}{5}(K - 273.16) + 491.69$

TABLE II²
ITcal g⁻¹ UNIT CONVERSIONS

$$1.0 \text{ International Steam Tables Calorie (ITcal)} = 1.00032 \text{ cal}_{15}$$

$$1.0 \text{ ITcal g}^{-1} = 4.18684 \times 10^7 \text{ cm}^2 \text{ sec}^{-2}$$

$$= 4.18684 \times 10^3 \text{ m}^2 \text{ sec}^{-2}$$

$$= 4.5057 \times 10^4 \text{ ft}^2 \text{ sec}^{-2}$$

¹The conversions given here are based on the values provided by List [5, p. 17].

²The conversions given here are based on the values provided by List [5, p. 14].

$$L_s = 98.777 + [335566.953 - |T - 238|^2]^{\frac{1}{2}} \quad (3)$$

The latent heat associated with the solid-liquid/liquid-solid phase changes is the latent heat of fusion, L_f . It can be approximated from

$$L_f = (1.7757 - 0.0023245T)T - 0.012 |T-243| - 231.494 \quad (4)$$

The accuracy of L_v , L_s , and L_f is given in Table III.

Saturation Vapor Pressures

In meteorological parlance, saturation vapor pressure refers to the partial pressure of water vapor for saturated conditions over a plane surface of either liquid or solid water. The saturation vapor pressure, expressed in millibars (conversion to other pressure units is provided in Table IV), is a function of temperature only.

The saturation vapor pressure associated with a plane liquid water surface, e_{s_w} , is approximated by

$$e_{s_w} = 10^{(a+b+c+d+e)} \quad (5)$$

where $a = 23.83224$

$b = -5.02808 \log_3 T$

$c = 8.1328 \times 10^{-7} \times 10^{(3.49142 - 1302.8844/T)}$

$d = -1.3816 \times 10^{-7} \times 10^{(11.344 - 0.0393998 T)}$

$e = -2949.076/T$

and the saturation vapor pressure associated with a plane solid water surface, e_{s_i} , is approximated by:

³ Equations 5 and 6 are restatements of those given by List [5, p. 350].

TABLE III⁴
 PERCENTAGE ERRORS IN THE NUMERICAL APPROXIMATIONS FOR CALCULATION
 OF LATENT HEATS

Temperature (C)	Percentage Errors in Latent Heats		
	Vaporization	Sublimation	Fusion
60	0.009		
55	0.004		
50	0.000		
45	-0.004		
40	0.009		
35	0.004		
30	0.017		
25	0.030		
20	0.043		
15	0.048		
10	0.051		
5	0.055		
0	0.074	0.000	0.063
-10	0.083	0.003	0.108
-20	0.057	-0.005	-0.080
-30	0.000	0.005	-0.246
-40	-0.153	0.005	-0.462
-50	-0.445	-0.005	0.350
-60		0.003	
-70		0.000	
-80		0.001	
-90		0.006	
-100		0.000	

⁴The comparative base for construction of this Table was that provided by List [5, p. 343].

TABLE IV⁵
PRESSURE CONVERSIONS

1.0 millibars (mb) = $10^3 \text{ g cm}^{-1} \text{ sec}^{-2}$
$1.4504 \times 10^{-2} \text{ lb in}^{-2}$
$2.08854 \text{ lb ft}^{-2}$

TABLE VI⁶
PERCENTAGE ERRORS IN THE NUMERICAL APPROXIMATIONS FOR CALCULATION
OF SATURATION VAPOR PRESSURE AT SELECTED TEMPERATURES

Temperature (C)	Percentage Errors	
	e_{s_w}	e_{s_i}
-100		0.003
-80		-0.015
-60		0.031
-40	0.017	0.008
-20	-0.003	-0.036
0	-0.004	-0.008
20	-0.004	
40	-0.003	
60	-0.004	
80	-0.005	
100	-0.004	

⁵The conversions given here are based on the values provided by List [5, p. 13].

⁶The comparative base for the construction of this table was that provided by List [5, pp. 351-353 and 360-361].

$$e_{s_i} = 10^{(a+b+c+d)} \quad (6)$$

where $a = 2.0702$
 $b = -2484.99/T$
 $c = 3.56654 \log T$
and $d = -0.0032098 T$

The percentage accuracies of the functions of e_{s_w} and e_{s_i} are given in Table V.⁷

Dew Point Temperature

The dew point temperature, T_d , is that which unsaturated air, cooled at constant pressure with no change in the water vapor mixing ratio, would have at saturation. The process for approximating this temperature is an iterative one, with the basis of closure being the difference between the actual vapor pressure determined from the dry bulb temperature and relative humidity and the calculated vapor pressure of the dew point temperature. Assuming that the dry bulb temperature and the relative humidity are known, the calculations are accomplished as follows:

1. Select the accuracy desired in the vapor pressure differences. The resulting number of iterations required for various accuracies are provided in Table VI.
2. Calculate the saturation vapor pressure of the dry bulb temperature, e_s , using Eq. 5.

3. Calculate the desired vapor pressure at the dew point temperature, e_a , from the relationship

$$e_a = \frac{e_s \text{ rh}}{100} \quad (7)$$

4. Make an initial estimate of the dew point temperature. The initial estimate in this report is found from a restatement of an approximation given by Tewson and Longley [6, p. 226]:

⁷ Applying the function statements stated here to all the values provided by List [5, pp. 351-353; 360-361] the maximum percentage error found was 0.04 percent.

TABLE VI⁸PERCENTAGE ERROR OF THE DEW POINT AND WET BULB TEMPERATURE ITERATION TECHNIQUE⁹

DEW POINT TEMPERATURE				WET BULB TEMPERATURE			
Average Computational Error	Computational RMS	Number of Iterations	Preset Allowable Error	Average Computational Error	Computational RMS	Number of Iterations	Pressure (mb)
		Average RMS	(vapor pressure)		Average RMS	Average RMS	
0.15	0.59	3	1	-0.04	0.53	6	3
-0.0025	0.053	8	3	-0.002	0.052	11	4
-0.00033	0.0052	13	3	-0.00037	0.0053	16	4
0.00003	0.00053	18	3	0.0001	0.00058	21	5
-0.00000	0.00005	23	3	-0.00000	0.00005	26	5
 <i>(a)</i>							
0.15	0.59	3	1	-0.04	0.53	6	3
-0.0025	0.053	8	3	-0.002	0.052	11	4
-0.00033	0.0052	13	3	-0.00037	0.0053	16	4
0.00003	0.00053	18	3	0.0001	0.00058	21	5
-0.00000	0.00005	23	3	-0.00000	0.00005	26	5
 <i>(a)</i>							
1.0	-0.03	0.54	6	3	600		
0.1	-0.0011	0.053	10	4			
0.01	-0.0002	0.0052	16	4			
0.001	-0.00001	0.00099	20	4			
0.0001	-0.00000	0.00006	25	4			

⁸This table was constructed on the basis of 1150 values for each preset allowable percentage error in the final vapor pressure. Temperatures used ranged from -10C to +10C at steps of 5C combined with relative humidities ranging from 1 percent to 99 percent at steps of 2 percent.

⁹Accuracy is expressed in terms of vapor pressure percentage difference between that resulting from the iterative process and that from the relative humidity (see Eqs. (10) and (15)).

$$T_d = \frac{2340}{8.573 - \log(0.16367e_a)} \quad (8)$$

5. Establish an initial increment for temperature iteration. The initial increment of temperature, ΔT , used in this report is determined from

$$\Delta T = 7.7 - 0.026T_d \quad (9)$$

with the added condition that $|\Delta T| > 0.1$.

6. Calculate the saturation vapor pressure corresponding to the estimated T_d , $e_s(T_d)$, using Eq. 5 of Table V.
7. Find the error in this vapor pressure from

$$\text{error} = \frac{100 [e_a - e_s(T_d)]}{e_a} \quad (10)$$

If this error is smaller than the value specified in Step 1, STOP: the T_d currently being investigated is acceptable as an estimate of the dew point temperature.

8. If the sign of the error has changed from the previous one and the error is larger than the value specified in Step 1, change ΔT to $\frac{1}{2}\Delta T$.
9. Adjust the estimated value, T_d , according to

$$T_d = T_d + \frac{e_a - e_s(T_d)}{|e_a - e_s(T_d)|} \Delta T \quad (11)$$

and then RETURN TO STEP 6.

The accuracy of the dew point temperature is dependent on the accuracy of Eq. 5 and the temperature of the dew point. With a closure accuracy requirement of 0.001 percent, the maximum error to be expected would range from 0.005 C at -50 C to 0.002 C at +100 C.

The percentage accuracies, exclusive of consideration of the vapor pressure function contributions, are provided in Table VI.

Wet Bulb Temperature

The wet bulb temperature, T_w , is that which unsaturated air, cooled by the adiabatic evaporation of water at constant pressure with no external source of heat, would have at saturation. Like the dew point temperature of the preceding section, the process for approximating this temperature is an iterative one, with the basis of closure being the difference in vapor pressures. Assuming that the dry bulb temperature and relative humidity are known, the calculations are accomplished as follows:

1. Select the accuracy desired in the vapor pressure differences. The resulting number of iterations required for various accuracies are provided in Table VI.
2. Calculate the saturation vapor pressure of the dry bulb temperature, e_s , using Eq. 5.
3. Calculate the vapor pressure at the dew point temperature, e_a , from the relationship stated in Eq. 7.
4. Make an initial estimate of the wet bulb temperature. The initial estimate in this report is found from a consideration of the dew point temperature, T_d , as estimated in Eq. 8, in the form

$$T_w = \frac{e_a T_d + e_s T_d}{e_a + e_s} \quad (12)$$

5. Make an initial estimate of the saturation vapor pressure of the dew point temperature, e_w , using the expression from List [5, p. 366] in the form

$$e_w = e_a + 0.00065 P(1 + 0.0011 T_w)(T_a - T_w) \quad (13)$$

6. Establish an initial increment for temperature iteration. The initial increment of temperature, ΔT , used in this report is determined from

$$\Delta T = [e_s(T_w) - e_w](3.3 - 0.0089T_a) \quad (14)$$

where $e_s(T_w)$ is found from Eq. 5, with the added condition that $|dT| > 0.2$.

7. Calculate the saturation vapor pressure corresponding to the estimated T_w , $e_s(T_w)$, using Eq. 5.
8. Calculate the vapor pressure, e_w , using Eq. 13.
9. Find the error between the two vapor pressures (from Steps 7 and 8) from

$$\text{error} = \frac{[e_s(T_w) - e_w] \cdot 100}{e_w} \quad (15)$$

If this error is smaller than the value specified in Step 1, STOP: the T_w currently being investigated is acceptable as an estimate of the wet bulb temperature.

10. If the sign of the error has changed from the previous one and the error is larger than the value specified in Step 1, change ΔT to $\frac{1}{2}\Delta T$.
11. Adjust the estimated value, T_w , according to

$$T_w = T_w + \Delta T \quad (16)$$

where ΔT is positive if $e_s(T_w) < e_w$ and negative if $e_s(T_w) > e_w$. RETURN TO STEP 7.

As for the dew point temperature, the accuracy of the wet bulb temperature is mainly dependent on the accuracy of Eq. 5 and the temperature of the wet bulb.

Specific Heat of Liquid Water

The amount of heat required to raise one gram of liquid water one degree K is the specific heat of water, c_w . The function for estimating the value of c_w results from a curve fitting of the data given by List [5, p. 343]. The function is

$$c_w = 0.9979 + \frac{0.3021}{2(8.5 - 0.1 |T - 308|)} \quad (17)$$

The accuracy of the application of this function with the values from List [5, p. 343] is summarized in Table VII.

METEOROLOGICAL PROCESS PARAMETERS

The meteorological parameters described in this section are associated with meteorological processes. The parameters covered are: (1) the temperature air would have after it has been lifted (dry adiabatically with no change in water vapor mixing ratio) from an initial level until saturation is reached, which corresponds to the temperature of the lifting condensation level (LCL); (2) the time rate of change of the mass of freely falling water drops; and (3) the temperature resulting from moving air from one pressure level to another in a maintained saturated state, which is the pseudo-adiabatic temperature.

Temperature of the LCL

The temperature of the LCL, T_{LCL} , is computed from the dry bulb temperature, T_a , and the dew point temperature, T_d , of the initial level in a restatement of the formulation provided by Inman [7], namely

$$T_{LCL} = T_d + (0.098 - 0.001571 T_d + 0.000436 T_a) (T_a - T_d) \quad (18)$$

TABLE VII

PERCENTAGE ERROR IN THE NUMERICAL APPROXIMATIONS FOR THE
CALCULATION OF THE SPECIFIC HEAT OF LIQUID WATER

Temperature (C)	Percentage Error of c_w
-50	0.000
-40	0.785
-30	-0.609
-20	-0.417
-10	-0.316
0	-0.006
5	0.087
10	0.132
15	0.144
20	0.146
25	0.127
30	0.108
35	0.084
40	0.108
45	0.137
50	0.176
55	0.234
60	0.322

The accuracy of the approximation is given in Table VIII, where the comparative base is that provided by Nordquist [8]. These values were computed using an iterative scheme, which had a closure of 0.001 percent in determination of the LCL water vapor mixing ratio.

Change of Mass of Freely Falling Water Drops

The construction of numerical models which contain consideration of liquid water require that some method be established for considering the mechanism of evaporation. The parameterization used here is based on the method proposed by Kessler [9, pp. 29-30] using the data provided by Kinzer and Gunn [10]. The basis of the approximation is that the change in mass due to evaporation can be expressed in the form

$$\frac{dM}{dT} = A (e_s - e_a) d^B \quad (19)$$

The two approximations provided here are based on "best" fits of the Kinzer and Gunn data to Eq. (19), one as the equation stands and the other including a temperature dependence. The first approximation is

$$\left. \frac{dM}{dT} \right|_1 = 0.0152 (e_s - e_a) d^{1.615} \quad (20)$$

while the second is

$$\left. \frac{dM}{dT} \right|_2 = [1. - 0.025(T_a - 290)] \left. \frac{dM}{dT} \right|_1 \quad (21)$$

The accuracy of these approximations for selected conditions is given in Table IX, with the Kinzer and Gunn data as the comparative base.

TABLE VIII
ERROR (C) OF THE LIFTING CONDENSATION LEVEL
TEMPERATURE FUNCTION

Initial Temperature (C)	Initial Relative Humidity (%)				
	90	70	50	30	10
-20	-0.015	-0.015	-0.013	0.008	0.045
0	0.007	0.007	0.005	-0.002	0.009
+20	0.013	0.014	0.011	-0.004	-0.038
+40	0.021	0.029	0.030	0.004	-0.072

TABLE IX¹⁰

PERCENTAGE ERROR OF THE dM/dt APPROXIMATIONS FOR FREELY
FALLING WATER DROPS FOR SELECTED CONDITIONS

Drop Diameter (cm)	Temperature				Relative Humidity (%)	
	0 C		40 C			
	$(dM/dt)_1$	$(dM/dt)_2$	$(dM/dt)_1$	$(dM/dt)_2$		
0.02	50	29	-43	40	10	
	45	29	-84	38	50	
	44	28	-90	36	90	
0.08	33	4	-96	17	10	
	33	5	-102	15	50	
	32	3	-109	12	90	
0.20	32	3	-106	13	10	
	32	4	-112	11	50	
	31	2	-119	8	90	
0.30	43	18	-84	22	10	
	43	19	-89	20	50	
	42	17	-96	17	90	
0.36	50	29	-64	31	10	
	50	29	-63	29	50	
	49	28	-4	27	90	

¹⁰While these percentage errors are quite large, they relate to rather small numbers, and are adequate for those problems where the cloud physics are treated in a gross sense.

Saturation-Adiabatic Temperatures

Air moved adiabatically from one pressure level to another in a saturated state can take on one of two different values, depending on the disposition of the water content of the air. If the total water content remains constant, the process is reversible and the resulting temperature is the saturation-adiabatic temperature, T_s . If the total water content decreases by the amount of liquid water present so that the process is irreversible, the resulting temperature is the pseudo-adiabatic temperature, T_m .

The technique for determining these temperatures requires use of the statement of the First Law of Thermodynamics, based on the definitions provided by Beers [11, p. 359], in the form

$$c_p dT - \frac{RT}{(P_t - e_s)} d(P_t - e_s) + T d\left(\frac{L_v X_s}{T}\right) + c_w (X_s + X_w) dT = 0 \quad (22)$$

where

$$X_s = \frac{\xi e_s}{(P_t - e_s)} \quad (23)$$

and the Clausius-Clapeyron Equation is

$$de_s = \frac{\xi L_v e_s}{RT^2} dT \quad (24)$$

Differentiation of Equations (2) and (23), and substitution along with Equation (24) into Equation (22), yields after rearrangement

$$\frac{dT}{dP_t} = \frac{RT_s + L_v X_s}{(P_t - e_s) \left(c_p + c_w (X_s + X_w) - 0.575 X_s + \frac{X_s^2 L^2 P_t}{RT^2 e_s} \right)} \quad (25)$$

To establish a value for the saturation-adiabatic temperature, it is assumed that no hydrometeors are within the air at the start of the process, so that $(X_s + X_w) = X_s(T_o, P_o) = X_z$. Thus

$$\frac{dT_s}{dP_t} = \frac{RT_s + L_v X_s}{(P_t - e_s) \left(c_p + c_w X_z - 0.575 X_s + \frac{X_s^2 L^2 P_t}{RT^2 e_s} \right)} \quad (26)$$

For the psuedo-adiabatic temperature, it is assumed that all the liquid water is removed from the air, so that $X_w = 0$. Thus

$$\frac{dT_m}{dP_t} = \frac{RT_m + L_v X_s}{(P_t - e_s) \left(c_p + c_w X_s - 0.575 X_s + \frac{X_s^2 L^2 P_t}{RT^2 e_s} \right)} \quad (27)$$

For many meteorological problems it is sufficient to assume that the partial pressure exerted by the air is much larger than that exerted by the water vapor included in the air, that the relative mass of the water vapor is small compared to the mass of the air, and that the latent heat of vaporization is constant. With these assumptions, Eq. (22) can be expressed as

$$c_p dT - \frac{RT}{P_t} dP_t + L_v dX_s = 0 \quad (28)$$

and Eq. (23) can be expressed as

$$x_s = \frac{\epsilon e_s}{P_t} \quad (29)$$

Here T_x is used to designate this meteorological approximation to the pseudo-adiabatic temperature

$$\frac{dT_x}{dP_t} = \frac{RT_x + L e_s}{P_t \left(C_p + \frac{x_s^2 L^2 P_t}{R T_e^2 e_s} \right)} \quad (30)$$

where for each Equation (26), (27), and (30), e_s and L are assumed to be functions of T_s , T_m , or T_x , as appropriate.

The solution of these equations is sought by using a second-order predictor-corrector method [12, pp. 385-386]¹¹, where the initial step is accomplished using

$$T_1 = T_0 + \left. \frac{dT}{dP} \right|_0 \Delta P \quad (31)$$

Thereafter, a first approximation "predictor" is made using

$$T_{i+1} = T_{i-1} + 2 \left. \frac{dT}{dP} \right|_i \Delta P \quad (32)$$

and the "corrector" is

¹¹A systems sub-program for the Univac 1108 computer is available for accomplishing this integration in a more general manner [18, sec. 11, p.1].

$$T_{i+1}^n = T_i + \frac{1}{2} \left[\left(\frac{\partial T}{\partial P} \right)_i + \left(\frac{\partial T}{\partial P} \right)_{i+1} \right] \Delta P \quad (33)$$

The "corrector" is repeated until

$$\frac{(T_{i+1}^n - T_{i+1}^{n-1})}{T_{i+1}^n} \leq \text{specified closure} \quad (34)$$

To provide some indication of the validity of this integration technique, the expression

$$\frac{dx}{dy} = 0.5 y^{-0.5} + 1 \quad (35)$$

was integrated from $y=1$ to $y=30$, with a Δy of 0.01 and a closure requirement of 10^{-3} . The results are given in Table X. While an accuracy analysis of the saturation- and pseudo-adiabatic temperatures is not possible due to a lack of a suitable comparative base, it is possible to compare the results with those obtained by another approximation method [5, p. 318]. The comparisons are given in Table XI and include the values provided by List [5, pp. 320-321] for pseudo-adiabatic temperatures with initial conditions of 1096.7 mb and 14C, program requirements of $\Delta p = 2$ mb, and a closure requirement of 10^{-3} .

TABLE X

COMPARISON OF THE RESULTS OF THE NUMERICAL METHOD FOR SOLVING A
DIFFERENTIAL EQUATION APPLIED TO $dx/dy = \frac{1}{3} y^{\frac{3}{2}} + 1$

Value of y	Initial Value of y	Value of x		Difference of Values of x Percentage
		Integration	$x = y^{\frac{1}{2}} + y$	
2	1	3.41421	3.41421	-0.00004
4	1	6.00000	6.00000	-0.00006
6	1	8.44949	8.44949	-0.00006
8	1	10.82843	10.82843	-0.00006
16	1	20.00000	20.00001	-0.00005
25	1	30.00000	30.00001	-0.00004
30	1	35.47723	35.47723	-0.00002

TABLE XI
COMPARISONS OF SATURATION ADIABATIC TEMPERATURES (C)

PRESSURE	PSEUDO-ADIASTATIC		SATURATED-ADIASTATIC	
	LIST	CALCULATED	CALCULATED	NET APPROX
1096.7	14.0	14.0	14.0	14.0
1046.7	12.0	12.0	12.0	12.0
1000.0	10.0	10.1	10.0	10.0
957.0	8.0	8.1	8.1	8.1
915.0	6.0	6.1	6.1	6.1
876.0	4.0	4.2	4.1	4.1
840.0	2.0	2.2	2.2	2.1
805.0	.0	.2	.1	.1
773.0	-2.0	-1.7	-1.8	-1.9
744.0	-4.0	-3.6	-3.7	-3.8
715.0	-6.0	-5.6	-5.7	-5.8
688.0	-8.0	-7.5	-7.7	-7.8
663.0	-10.0	-9.4	-9.6	-9.7
638.0	-12.0	-11.4	-11.7	-11.8
616.0	-14.0	-13.3	-13.6	-13.7
595.0	-16.0	-15.2	-15.5	-15.6
573.0	-18.0	-17.2	-17.6	-17.8
554.0	-20.0	-19.1	-19.6	-19.7
536.0	-22.0	-20.9	-21.5	-21.6
518.0	-24.0	-22.9	-23.5	-23.6
502.0	-26.0	-24.7	-25.3	-25.5
485.0	-28.0	-26.7	-27.4	-27.6
470.0	-30.0	-28.5	-29.3	-29.5
454.0	-32.0	-30.6	-31.4	-31.6
440.0	-34.0	-32.4	-33.4	-33.5
426.0	-36.0	-34.4	-35.4	-35.5
413.0	-38.0	-36.2	-37.3	-37.5
400.0	-40.0	-38.1	-39.3	-39.4
387.5	-42.0	-40.0	-41.3	-41.4

TABLE XI (cont.)
COMPARISONS OF SATURATION-ADIABATIC TEMPERATURES (C)

PRESSURE	PSEUDO-ADIASTIC		SATURATED-ADIASTIC	
	LIST	CALCULATED	CALCULATED	NET APPROX
375.3	-44.0	-42.0	-43.3	-43.4
363.5	-46.0	-43.9	-45.3	-45.4
352.1	-48.0	-45.8	-47.2	-47.4
341.0	-50.0	-47.7	-49.2	-49.4
330.2	-52.0	-49.6	-51.2	-51.4
319.7	-54.0	-51.5	-53.2	-53.4
309.5	-56.0	-53.4	-55.2	-55.4
299.6	-58.0	-55.3	-57.2	-57.3
289.9	-60.0	-57.2	-59.2	-59.3
280.5	-62.0	-59.1	-61.2	-61.3
271.3	-64.0	-61.0	-63.1	-63.3
262.3	-66.0	-62.9	-65.1	-65.3
253.6	-68.0	-64.7	-67.1	-67.3
245.1	-70.0	-66.6	-69.1	-69.3
236.8	-72.0	-68.5	-71.1	-71.3
228.7	-74.0	-70.3	-73.1	-73.2
220.8	-76.0	-72.2	-75.1	-75.2
213.2	-78.0	-74.0	-77.1	-77.2
205.7	-80.0	-75.8	-79.1	-79.2
198.4	-82.0	-77.7	-81.1	-81.2
191.3	-84.0	-79.5	-83.0	-83.2
184.3	-86.0	-81.3	-85.0	-85.2
177.6	-88.0	-83.0	-87.0	-87.2
171.0	-90.0	-84.8	-89.0	-89.2
164.6	-92.0	-86.5	-91.0	-91.2
158.4	-94.0	-88.3	-93.0	-93.2
152.4	-96.0	-89.9	-95.0	-95.1

CONCLUDING REMARKS

This report has presented numerical techniques for approximating the values of several meteorological parameters along with the accuracies of the results of the approximations. This has been done so that (1) the methods might be documented for use by others, (2) the representativeness of the results of applying the techniques might be made available for analysis for other purposes, and (3) the comparison of these techniques with others can be readily accomplished.

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APPENDIX

COMPUTER LANGUAGE LISTINGS OF FUNCTIONS AND SUBROUTINES

PAGE |

NUMERICAL APPROXIMATIONS

+++ FUNCTION STATEMENTS +++

TX = DRY BULB TEMPERATURE (K)
TZ = DEW POINT TEMPERATURE (K)
ES SATURATION VAPOR PRESSURE (MB)
EA ACTUAL VAPOR PRESSURE (MB)
DIA WATER DROP DIAMETER (M)

.....LATENT HEAT OF VAPORIZATION.....

HEATV(TX)= 754.817 - 0.575*TX

.....LATENT HEAT OF SUBLIMATION.....

HEATS(TX)= 98.777 +SQRT(335566.953 - ABS(TX - 238.)*².)

.....LATENT HEAT OF FUSION.....

HEATF(TX)=TX*(1.7757-0.0023245*TW)-0.012*ABS(TX-243.)-231.494

.....SATURATION VAPOR PRESSURE OVER WATER.....

SATVP(TX)=10.*²(23.83224-2949.076/TX -5.02808*ALOG10(TX)
| -1.3816E-7*10.*²(11.3440 - 0.0303998*TX)
2 +8.1328E-3*10.*²(3.49149 - 1302.8844/TX))

.....SATURATION VAPOR PRESSURE OVER ICE.....

SATVI(TX)=10.*²(2.0702-2484.991/TX+3.56654*ALOG10(TX)-.0032098*TX)

7

PAGE 2

NUMERICAL APPROXIMATIONS

..... TEMPERATURE OF THE LIFTING CONDENSATION LEVEL.....

$$TLCL(TX, TZ) = TZ + (0.098 - 0.001571 * TZ + 0.000436 * TX) * (TX - TZ)$$

..... SPECIFIC HEAT OF LIQUID WATER.....

$$SPH20(TX) = 0.9979 + 0.3021 / (2. ** (8.5 - 0.1 * ABS(TX - 308.16)))$$

..... DM/DT FOR FREELY FALLING WATER DROPS.....

$$DMDT(ES, EA, DIA) = 0.0152 * (ES - EA) * DIA * 1.615$$

$$DMDTT(ES, EA, DIA, TX) = DMDT(ES, EA, DIA) * (1. - 0.025 * (TX - 290.))$$

8

10

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SUBROUTINE DEWPT(T, RH, ET, TD)

12

```

ESTIMATING DEW POINT TEMPERATURES          12
      ACTUAL VAPOR PRESSURE -ESTIMATE        MB
      ACTUAL VAPOR PRESSURE        MB
      SATURATION VAPOR PRESSURE        MB
      ALLOWABLE PERCENTAGE ERROR    PERCENT
      TEMPERATURE INCREMENT          K
      RELATIVE HUMIDITY             PERCENT
      TEMPERATURE - DRY BULB       K
      TEMPERATURE - DEW POINT      K

SATVTP ( TX)=10.** (23.83224 -2949.076/TX           13
                  -5.02808*ALOG10(TX)
                  -1.3816E-7*10.** (11.344-.0303998*TX)   14
                  +8.132E-3*10.** (3.49149-1302.8844/TX)     15
                                         )                         16

ES=SATVTP (T)                                     17
EA=RH*ES/100.                                      18
C =0                                              19
TD=2340./ (8.573-ALOG10(0.16367*EA))           20
IF (RH.GE.100.) TD=T                               21
DEL=.7-.026*TD                                     22
IF (ABS(DEL).LT.0.1) DEL=0.                         23
E=SATVTP (TD)                                     24
D=(E-EA)*100./EA
IF (ABS(D).LT.ET) RETURN
G=C*D
IF (G.LT.0.) DEL=DEL*0.5
R=.00.*E/ES
TD=TD+(RH-R)*ABS(DEL/(RH-R))
C=D
GO TO 1
END

```

SUBROUTINE WETBLB(T,P,RH,ET,TW)

ESTIMATING WET BULB TEMPERATURE

```

DEL      TEMPERATURE INCREMENT          K
E       ACTUAL VAPOR PRESSURE -ESTIMATE   MB
EA      ACTUAL VAPOR PRESSURE -WET BULB   MB
EB      ACTUAL VAPOR PRESSURE -DEW POINT   MB
ES      SATURATION VAPOR PRESSURE        MB
ET      ALLOWABLE PERCENTAGE ERROR      MB
P       PRESSURE                         PERCENT
RH     RELATIVE HUMIDITY                 K
T       TEMPERATURE - DRY BULB           K
TW     TEMPERATURE - WET BULB           K

SATVP(TX)=10.**(23.83224 -2949.076/TX)          35
|           -5.02808* ALOG10(TX)                   36
|           -1.3816E-7*T*10.**(11.344-.0303998*TX) 37
2           +8.1328E-3*T*10.**(3.49149-1302.8844/TX)) 38
3           ALIST(PX,DX,TX)=0.00066*(1.+0.00115*TX)*PX*Dx 39

ES=SATVP(T)
EA=RH*EX/100.                                     40
EB=EA
TB=EA
TB=2340./ (8.573-ALOG10(0.16367*EA))          41
TB=(TW*ES+T*EA)/(EA+ES)                           42
IF (RH.GE.100) TW=T                                43
DE=T-TW
EK=EA+ALIST(P,DE,TW)                               44
DEL=(SATVP(TW)-EK)*(3.3-.009*T)                  45
IF (ABS(DEL).LT.0.2) DEL=0.2                      46
E=SATVP(TW)
DE=T-TW
EA=EB+ALIST(P,DE,TW)                               47
D=(E-EA)*100./EA                                  48
49
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PAGE 5 NUMERICAL APPROXIMATIONS

```
IF (ABS(D).LT.ET) RETURN  
G=C*D  
DEL=ABS(DEL)  
IF (G.LT.0.) DEL=DEL*.5  
IF (EA.LT.E) DEL=DEL  
TW=TW+DEL  
C=D  
GO TO 1  
END
```

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PAGE 6

NUMERICAL APPROXIMATIONS

SUBROUTINE ADIAB(T1,P1,P2,L,TSAT)

TO CALCULATE SATURATION-ADIAZABATIC TEMPERATURE FROM
TEMPERATURE T1 AT PRESSURE P1 TO PRESSURE P2
FOR L=1 PSUEDO-ADIAZABATIC PROCESS
• 2 SATURATION ADIAZABATIC PROCESS
• 3 MET APPROXIMATION
•

REFERENCE INTEGRATION TECHNIQUE
RITGER AND ROSE
DIFFERENTIAL EQUATIONS WITH APPLICATIONS
(MCGRAW-HILL, 1968)
PP. 385-386

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EXTERNAL DTDP FUNCTION REQUIREMENTS.....
SATVP(TX)=10.**(23.83224-5.02808*ALOG10(TX)-1.3816E-7*10.**(-11.344
1-.0303998*TX)+8.1328E-3*10.**(.3.49149-1302.8844/TX)-2949.076/TX)
COMMON/AA/XAI STORAGE REQUIREMENTS.....
DIMENSION TX(5),PX(5),TZ(20) INITAILIZE
ERRORT=1.E-3
I=1
IX=1
KJ=1
INITIAL CONDITIONS
XAI=SATVP(T1)
XAI=.62197*XAI/(P1-XAI)
PX(I)=P1
TX(I)=T1
DELDP=P2-P1
NEW INITIAL CONDITIONS
63
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DP= ABS(DELDP)*1.E-6/DELDP      78
PX(2)=PX(1)+DP                79
TX(2)=TX(1)+DP*DTDP(TX(1),PX(1),L) 80
PXREF=PX(2)                   81
IP=ABS(DELDP)*0.5              82
IF(IP.LT.1) IP=1              83
                                SUCCEEDING CONDITIONS
1 DO 5 K=IX,IP               84
DP=IP                         85
DP=(P2 -PXREF)/DP            86
IF (ABS(DP).LT.1.E-12) GO TO 7. 87
PX(I+2)=PX(I+1)+DP          88
DF=DTDP(TX(I+1),PX(I+1),L)  89
TX(I+2)=TX(I)+2.*DP*DF    90
TZ(I)=TX(I+2)                91
LL=L                         92
                                CLOSURE ITERATIONS
DO 2 M=2,20                  93
KI=KI+1                      94
MM=M                         95
DTDP2=DTDP(TZ(M-1),PX(I+2),L) 96
TZ(M)=TX(I+1)+0.5*DP*(DF+DTDP2) 97
TZR=ABS((TZ(M)-TZ(M-1))/TZ(M)) 98
IF (TZR.LT.ERRORT) GO TO 3    99
2 CONTINUE                     100
WRITE(6,998) TX(I+1),TZ(M-1),TZ(M),TZR,I,K,M 101
998 FORMAT(1H 4F10.5,3I6)      102
                                DID NOT CONVERGE HALVE DP
IX=LL                         103
IP=IP*2                        104
GO TO 6                         105
3 TX(I+2)=TZ(MM)               106
DO 4 N=2,3                      107
TX(N-1)=TX(N)                 108
PX(N-1)=PX(N)                 109
4

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PAGE 8

NUMERICAL APPROXIMATIONS

```
5 CONTINUE  
  TSAT=TX(I+2)  
  WRITE(6,999) KI  
999 FORMAT(1H+90X19)  
 RETURN  
6 GO TO 1  
7 WRITE(6,8) PX(I+2),TX(I+2),KI,DP,TI,P1,P2  
8 FORMAT(1H /29H THIS DID NOT CONVERGE AT P =F8.1,8H AND T =F9.2/IX  
1 3HON I5,24H ITERATIONS OF DELTA P =1PE12.5/IX  
2 13HGOING FROM T=OPPF8.1, 3H P=F8.1, 6H TO P=F8.1)  
 RETURN  
END
```

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NUMERICAL APPROXIMATIONS

FUNCTION DTDP(T,P,TX)

FUNCTION FOR THE CALCULATION OF THE CHANGE OF SATURATED-ADIASTATIC
TEMPERATURE WITH PRESSURE UNDER THREE BASIC SETS OF CONDITIONS

COMMON/AA/XI
.....STORAGE REQUIREMENTS.....

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FUNCTION REQUIREMENTS

HEATV(TX)=754.817-.575*TX
SATVP(TX)=10.**(23.83224-5.02808* ALOGIO(TX)-1.3816E-7*10.**(11.344
|-0.0303998*TX)+8.1328E-3*10.**(3.49149-1302.8844/TX)-2949.076/TX)

A=.068557*T
E=SATVP(T)
H=HEATV(T)
XA=XI
CX=-.575
TXX=ABS(T-380.16)
CW=0.9979+0.3021/(2.**(8.5-0.1*TXX))
PB=P-E
IF(IX.EQ.3) PB=P
X=0.62197*E/PB
CP=.2399+.4409*X
IF(IX.EQ.2) XA=X
IF(IX.EQ.3) XA=0.
IF(IX.EQ.3) CX=0.
AI=(A+H*X)/PB
BL=X*X*H*H*P/(A*T*E)
DTDP=AI / (CP+CW*X*A+CX*X+B1)
RETURN
END

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